

**FACTORS INFLUENCING THE RESIDENCE TIME DISTRIBUTION
IN CONTINUOUSLY-FED BATCH-TAPPED FURNACES**

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ABSTRACT

Residence time distributions are well defined for batch and continuously stirred reactors. However, there exist many smelting furnaces that do not fit either of these classifications. A mathematical description is presented for the case of a furnace that is fed continuously, but where the liquid slag and metallic products are tapped intermittently. The effects of slag and metal tapping intervals, and the fraction of liquid retained in the furnace, are shown to have a significant effect on the residence time distribution.

KEYWORDS

Pyrometallurgy, furnace, residence time distribution, continuous, batch tapped.



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INTRODUCTION

There are some processes in which the time spent by the feed material in a smelting furnace plays a significant role in determining the extent of the reactions between metal and slag, and influences the degree of settling of metal droplets from a molten slag.

Residence time distributions are well defined for batch and continuously stirred reactors. However, there exist many smelting furnaces that do not fit either of these classifications. Here we will focus on producing a mathematical description for the case of a furnace that is fed continuously, but where the liquid slag and metallic products are tapped intermittently. The effects of slag and metal tapping intervals, and the fraction of liquid retained in the furnace, are studied.

Previous work (Jones, la Grange, and Assis, 1997) has studied the effect of retention time on the degree of cobalt recovery achieved in a slag reduction process. In that work, a method was proposed for the calculation of the mean residence time of the contents of a pilot-plant furnace. That approach is to be extended here for the purposes of providing a more general design or scale-up tool.

The maximum allowable depth of molten material in a given furnace is affected by such factors as the desired batch size (or the size of the ladles) and the pressure of the molten material that the tap-hole and surrounding system can withstand. Within these constraints, there might be some scope for choice of tap-hole position. The effects of this choice need to be understood mathematically.

MATHEMATICAL DESCRIPTION

Simplifying Assumptions

In this paper, for the purpose of keeping the argument simple, we will be talking of the molten material in the furnace as though it was a single phase, and we are neglecting the separation of the feed into two streams that may be tapped separately. (As long as the amounts of metal are small, or if the metal and slag are tapped at similar times in similar ratios, this approximation is not a bad one. In other cases, the same methods could be applied, but the resulting equations would be somewhat more complicated.) We are also assuming, for clarity of discussion, that the slag is less dense than the metal, so that the slag floats on top of the metal, but this doesn't really affect the main argument at all.

We assume that the feed rate into the furnace is kept constant, *i.e.* each batch is fed perfectly evenly over the duration of each feeding period. The same quantity of material is fed during each cycle of feeding and tapping. After tapping, the quantity of molten material remaining in the furnace is kept the same every time. The material remaining in the furnace is essentially the portion of the molten bath below the level of the (slag) tap-hole. The furnace contents at the beginning of a tapping cycle are level with the tap-hole, and increase linearly in height during the cycle. Once the tap-hole is opened, the contents are drained down to the level of the tap-hole again. It is assumed that the molten slag and metal inside the furnace are each perfectly mixed.

Calculation Method

The method for calculating the mean residence time of a semi-continuous batch process is not standard textbook material, and so needs to be outlined here. Given the assumptions previously stated, it is possible to calculate the amount of feed from any particular tap number that is inside the furnace at any given time. From the tap-to-tap time, it is possible to obtain the mean residence time for each portion of feed in the furnace at a given time. Summing the product of the proportions of feed and the respective mean residence times gives the mean residence time for all material in the furnace at a particular time.

To illustrate the method of calculation more clearly, consider the following example, depicted in Figure 1. A set mass is fed into the furnace (at a constant feed rate) over a 2-hour time period. This mass

is considered to be the initial warm-up mass fed, *i.e.* no tapping occurs at the end of this stage. This mass has a mean residence time of 1 hour (with a residence time distribution that varies evenly from zero to two hours). The same set batch mass is again fed into the furnace (*i.e.* feed for tap 1), again at a constant feed rate, over 2 hours. Just before the furnace is tapped, the warm-up feed has a mean residence time of 3 hours, whilst the fresh material fed during tap 1 has a mean residence time of 1 hour. However, since the amount from each feed was identical, the mean residence time of any random particle removed from the furnace is 2 hours. Now, when the same set mass is fed during tap 2, this mass forms half of the total mass in the furnace, and has a mean residence time of 1 hour just before tapping. Also, at this point, the warm-up feed constitutes one quarter of the total mass in the furnace (mean residence time 5 hours) and the feed from tap 1 forms the remaining quarter of the total furnace mass (mean residence time 3 hours). In this case, the mean residence time of any random particle removed from the furnace is 2.5 hours. Similarly, it is possible to calculate the mean residence time of any particle at any other point in time, and the calculations can be continued until steady state has been reached. This steady-state distribution is the one of primary interest, as the effects of the first few tapping cycles are rather transient.

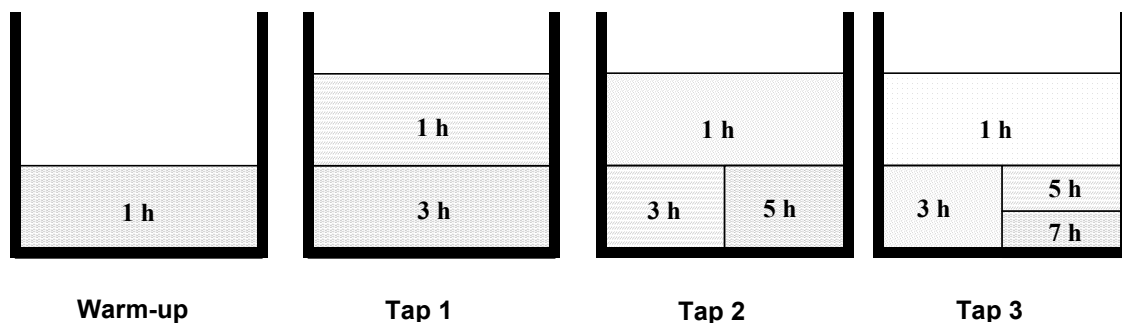


Figure 1 – Example showing how mean residence times are calculated at the end of each feeding cycle, just before the furnace is tapped

Abstraction of the Problem

In order to express this system mathematically, some definitions are required.

t = tap-to-tap time (hours)
 n = tap number
 $1/f$ = fraction removed during tapping

In considering residence time, one of the key variables in the design of a furnace is the position of the slag tap-hole, as this determines the fraction of molten material removed during tapping. Figure 2 shows the height (s) of the tap-hole above the hearth, in relation to the maximum height of the slag above the hearth level (h). Clearly, the maximum slag level is attained immediately before the furnace is due to be tapped. The fraction of slag removed during tapping ($1/f$) is shown, in Equation (1), as being equal to the amount of slag tapped divided by the sum of the slag tapped plus the slag remaining in the furnace.

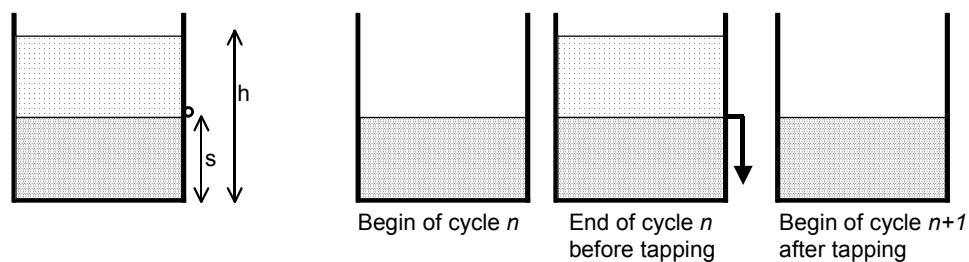


Figure 2 – Positions of slag level before (s) and after (h) tapping

$$\frac{1}{f} = \frac{h-s}{h} = 1 - \frac{s}{h} \tag{1}$$

Figure 3 shows the relative positions indicated by various values of f and $1/f$.

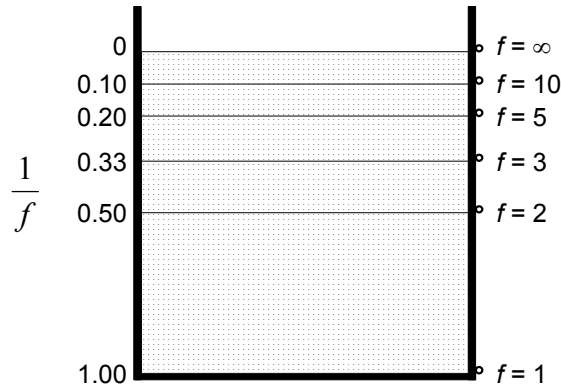


Figure 3 – Tap-hole positions indicated by various values of f

Example Where 1/2 of the Material is Removed During Tapping

Consider the case where one half of the molten material is removed from the furnace, and half is retained inside the furnace each time the furnace is tapped, and the tap-to-tap time is two hours. For the fresh material added during the most recent tapping cycle, the residence time will vary between zero and two hours, with a mean residence time of one hour. Just before the furnace is tapped, this fresh material (with a mean residence time of 1 hour) will make up half of the material inside the furnace. The remaining half will, in turn, have half of that material with a mean residence time two hours longer than that of the fresh material, and so on. This is illustrated diagrammatically in Figure 4.

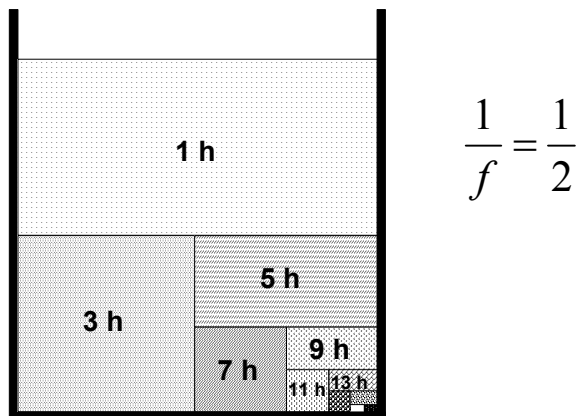


Figure 4 – A graphical depiction of the ‘steady-state’ residence time distribution, for a tap-to-tap time of 2 hours, with steady feeding, and half of the material being tapped from the furnace ($f = 2$)

The overall mean residence time may be expressed as:

$$\text{Mean residence time} = \left(\frac{1}{2} \times 1h\right) + \left(\frac{1}{4} \times 3h\right) + \left(\frac{1}{8} \times 5h\right) + \left(\frac{1}{16} \times 7h\right) + \dots \quad (2)$$

$$\text{Mean residence time (h)} = \frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{7}{16} + \frac{9}{32} + \frac{11}{64} + \frac{13}{128} + \frac{15}{256} + \frac{17}{512} + \frac{19}{1024} + \dots = 3 \quad (3)$$

This can be expressed more concisely as

$$\sum_{n=1}^{\infty} \frac{2n-1}{2^n} = 3 \quad (4)$$

Example Where 1/3 of the Material is Removed During Tapping

In this example, also with a tap-to-tap time of two hours, the residence time of feed material added continuously during that period varies between zero and two hours, with a mean residence time of one hour. The fraction of that material that is retained inside the furnace for the next two-hour tap period will have a mean residence time of an additional two hours, *i.e.* the mean residence time of that portion of material will be three hours. Figure 5 shows a graphical representation of the various fractions of the molten material and their respective mean residence times.

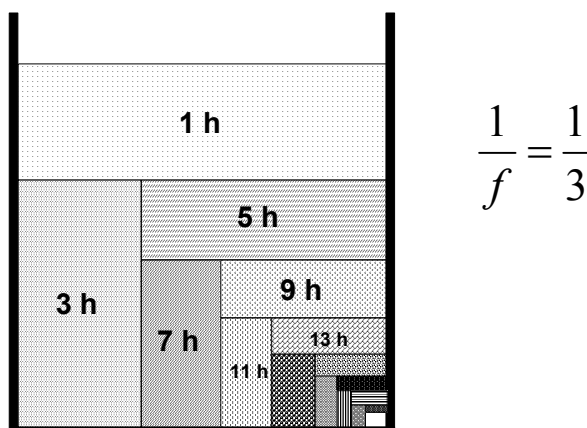


Figure 5 – A graphical depiction of the ‘steady-state’ residence time distribution, for a tap-to-tap time of 2 hours, and one third of the material being tapped from the furnace ($f=3$)

From the steady-state distribution depicted in Figure 5, it can be seen that the fraction of material that has been in the furnace for 1 hour is $1/3$. The fraction that has been in the furnace for 3 hours is $1/3 \times 2/3 = 2/9$. The remaining fraction that has been in the furnace for 5 hours or more is $(1 - 3/9 - 2/9) = 4/9$. Therefore, the fraction with a mean residence time of 5 hours is $1/3 \times 4/9 = 4/27$, and so on.

The overall mean residence time may be expressed as:

$$\text{Mean residence time (h)} = \left(1 \times \frac{1}{3}\right) + \left(3 \times \frac{2}{9}\right) + \left(5 \times \frac{4}{27}\right) + \left(7 \times \frac{8}{81}\right) + \left(9 \times \frac{16}{243}\right) + \dots = 5 \quad (5)$$

This can be expressed more concisely as

$$\sum_{n=1}^{\infty} (2n-1) \frac{2^{(n-1)}}{3^n} = 5 \quad (6)$$

Generalized Expression

The generalized expression for mean residence time may be written as shown in Equation (7), and simplified as follows:

$$\text{Mean residence time} = \sum_{n=1}^{\infty} t \left(n - \frac{1}{2} \right) \cdot \frac{(f-1)^{(n-1)}}{f^n} \quad (7)$$

$$= \frac{t}{f-1} \sum_{n=1}^{\infty} \left(n - \frac{1}{2} \right) \left(\frac{f-1}{f} \right)^n \quad (8)$$

$$= \frac{t}{f-1} \sum_{n=1}^{\infty} \left(n - \frac{1}{2} \right) \left(1 - \frac{1}{f} \right)^n \quad (9)$$

$$= \frac{t}{f-1} \left[\sum_{n=1}^{\infty} n \left(1 - \frac{1}{f} \right)^n - \frac{1}{2} \sum_{n=1}^{\infty} \left(1 - \frac{1}{f} \right)^n \right] \quad (10)$$

This can be simplified further by recognising that this is the sum of a hypergeometric series (of the form $\sum_n n C^n$) plus the sum of a geometric series (of the form $\sum_n C^n$).

$$= \frac{t}{f-1} \left[f(f-1) - \frac{f-1}{2} \right] \quad (11)$$

This equation simplifies down very tidily to a rather simple result.

$$\text{Mean residence time } R = t \left(f - \frac{1}{2} \right) \quad (12)$$

This result is shown graphically in Figures 6 and 7.

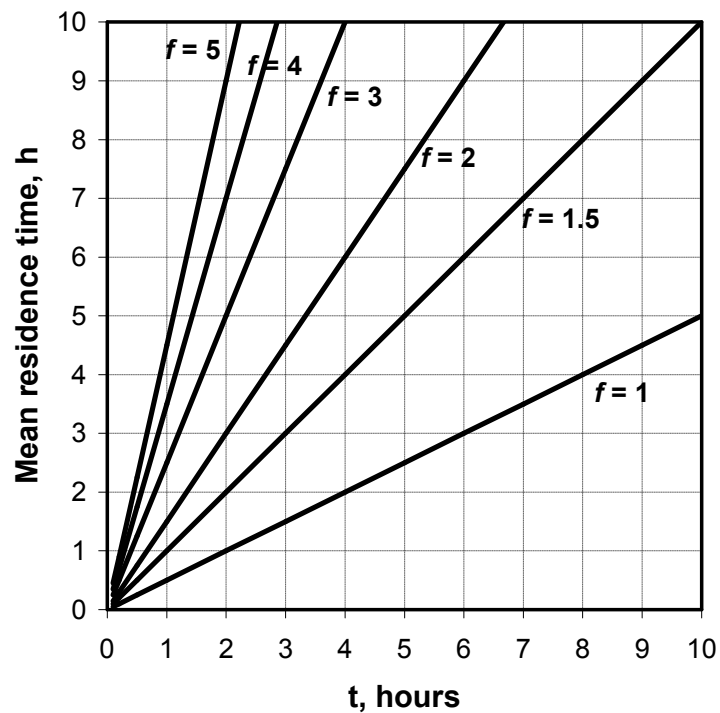


Figure 6 – Mean residence time shown as a function of tap-to-tap time t for a range of values of f

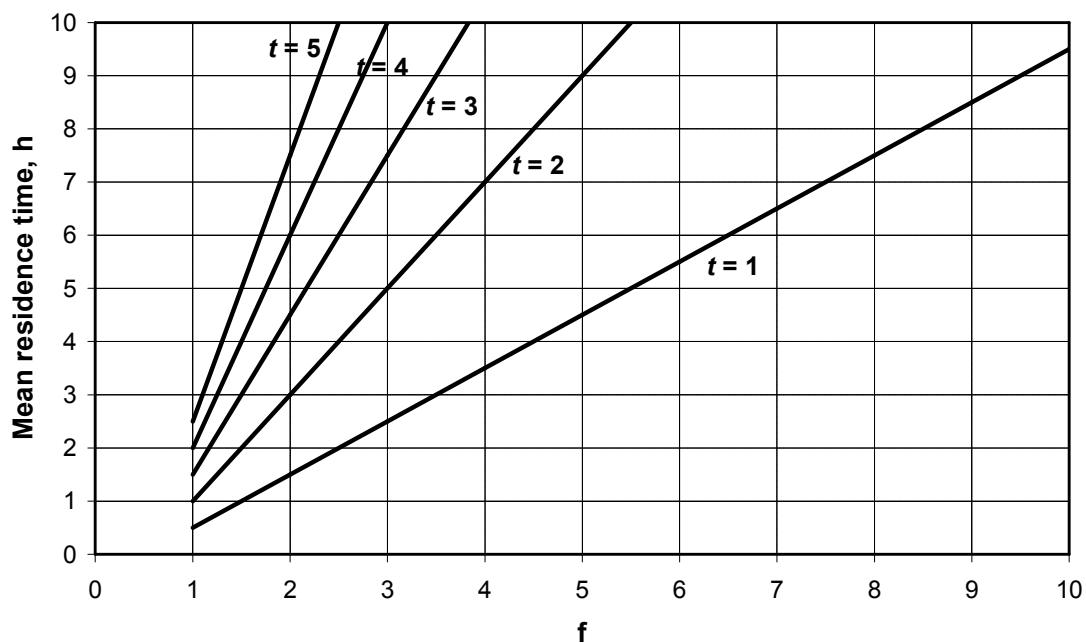


Figure 7 – Mean residence time shown as a function of f for a range of values of tap-to-tap time t

APPLICATIONS

Scale-up with Constant Residence Time

If one wants to achieve a constant mean residence time when scaling up from a pilot plant to a commercial plant, there is some flexibility in the choice between tap-hole position (indicated by f) and the tap-to-tap time, t . In Figure 8, curves of constant mean residence time (R) are shown as function of f and t .

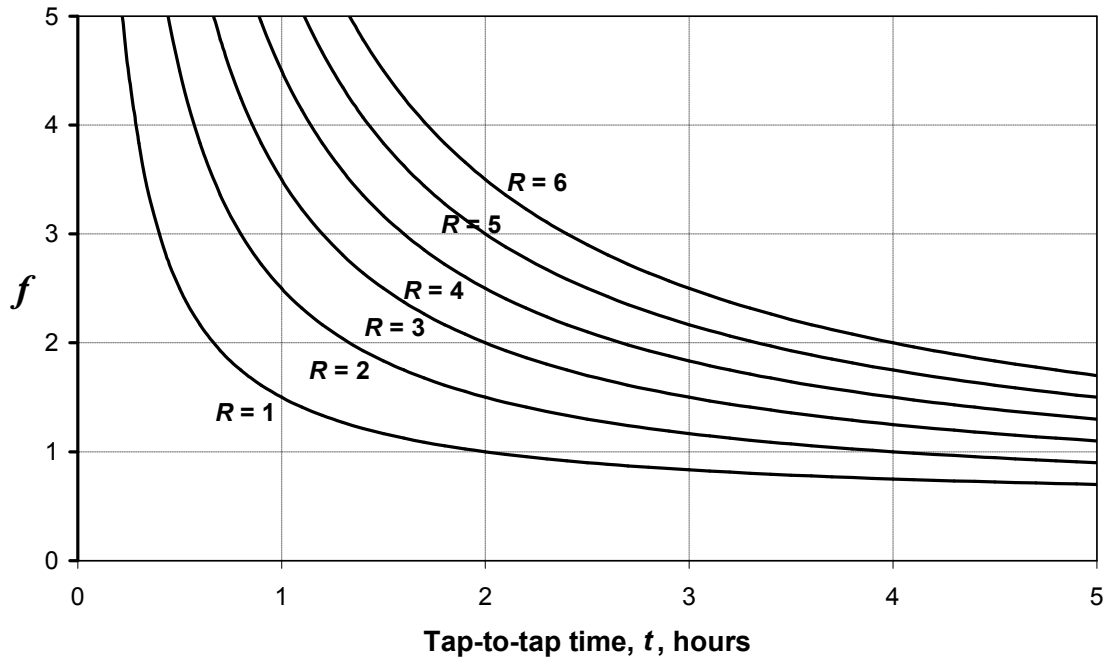


Figure 8 – Curves of constant mean residence time R (hours), as a function of f and t

An interesting observation from Figure 8 (and also Figure 6) is that the mean residence time (at steady-state) is equal to the tap-to-tap time when $f = 1.5$, *i.e.* when two thirds of the molten material is tapped from the furnace each time.

Number of Taps Needed to Change Composition

When operating a furnace, it is sometimes necessary to change composition of the molten bath. The question then arises as to how many taps it would take to substantially replace material of the old composition with that of the new. This number of taps depends strongly on the value of $1/f$, *i.e.* the fraction of material removed during tapping. This could be determined graphically by using diagrams similar to those in Figures 4 and 5, or algebraically using Equation (13).

$$\text{Residual fraction after } N \text{ taps} = 1 - \sum_{n=1}^N \frac{(f-1)^{n-1}}{f^n} \quad (13)$$

Figure 9 shows the fraction of 'old' material remaining inside the furnace after N taps. For example, if one wanted to ensure that the molten material inside the furnace comprises at least 90% 'new' material, then one would require at least 4 taps for $f=2$; at least 6 taps for $f=3$; and at least 11 taps for $f=5$.

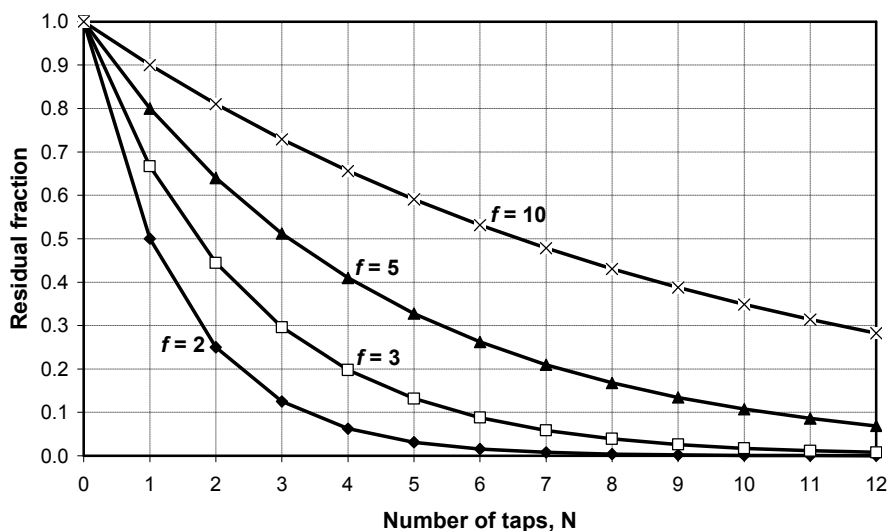


Figure 9 – The residual fraction of ‘old’ material left inside the furnace after N taps

CONCLUSIONS

It is clear that the mean residence time is increased by increasing the volume of material retained in the furnace. The mean residence time is directly proportional to the tap-to-tap time. Equations have been presented to allow one to calculate the mean residence time in a continuously-fed batch-tapped furnace.

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